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Comparison of approximation methods for predicting structural response of a honeycomb panel

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Abstract:

This paper discusses the findings of a study comparing different approximation models or machine learning models for prediction of structural responses of a honeycomb panel. Three different approximation models have been compared- Response Surface Model (RSM), Radial Basis Functions (RBF) and Universal Kriging method (UK). For each approximation model, average relative error between the predicted response and FEA response is compared for different designs. The findings are summarized and potential next steps to fortify the study are listed

Keywords: Surrogate models, Machine learning, Neural networks, approximation, black-box modeling, structural analysis, FEA

Introduction

Approximations or surrogate modeling is a subset of machine learning that could be used to predict an approximate response for different functions. This involves training and validation of approximation models based on a set of observed data points. Once the training is complete, these models can be used to predict a response at unobserved data points. Approximations find use in multitude of applications. One such application is structural analysis using Finite Element Methods (FEM).

Based on the application at hand, Finite Element Analysis (FEA) can be computationally very expensive and for certain complex analysis, compute time can range from hours to days depending on choice of compute architecture and scalability of the numerical problem at hand.

Approximations can be very useful in these situations. FEA could be run for limited number of data points and approximation methods can be used to build surrogate models which can predict the response for FEA datapoints that are originally unsolved for.

Approximation methods can be of different types and can provide different accuracies in prediction. This paper aims to compare three commonly used approximation methods- Response Surface Model, Radial Basis Function and Universal Kriging method.

Materials and Methods

Stress analysis of a honeycomb panel has been used as an example in this study. Honeycomb core sandwich panel is formed by adhering two, high-rigidity, thin face sheets with a low-density honeycomb core possessing less strength and stiffness [1]. Adhesively bonded sandwich structures, with their advantages of light weight, design flexibility, high specific stiffness and specific strength, are attractive structural components and are therefore widely used in aviation, space, and marine applications. The face-sheet (skin) and core of sandwich structures can encompass a myriad of materials, both composite and metallic [2]. [2] lists multiple combinations of such materials used in different applications. One such combination is metallic sandwich structure, which especially uses aluminum facesheet over aluminum honeycomb, is widely used as slat wedge, trailing edge, ailerons on aircraft, and in satellite structures [2]. This study uses structural analysis of such metallic sandwich structure. It is assumed that the entire structure including the face-sheet and core is made of Aluminum and generic mechanical properties are used for the stress analysis.

This study is done in multiple steps. Fig. 1. shows the steps used in the workflow:

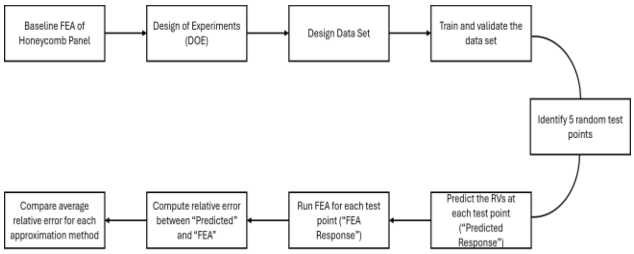


Fig. 1. Process Flowchart

Baseline FEA of a Honeycomb Panel

A Honeycomb Panel 400mm long and 150mm wide (Fig. 2) is considered. The face-sheets and the core are assumed to have generic Aluminum material properties with a Young's modulus of 7e4 MPa and Poisson's ratio of 0.3.

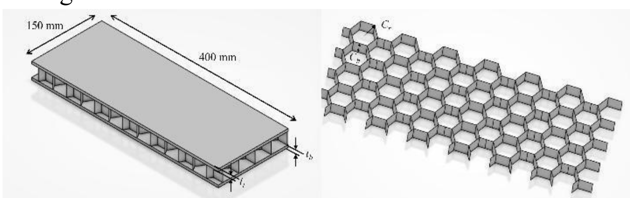


Fig. 2. Honey Comb Panel

The design is controlled by following geometric parameters:

- Cell size (C_r): controls the size of the regular hexagonal cell that makes up the core
- Cell Thickness (t_c): controls the thickness of the core
- Cell Height (C_h): controls the core height
- Top Sheet Thickness (t_t): controls the thickness of the top face-sheet
- Bottom Sheet Thickness(t_b): controls the thickness of the bottom face-sheet

The baseline values of geometric parameters is shown in TABLE I.

TABLE I. BASELINE VALUES OF GEOMETRIC PARAMETERS

C_r	t_c	C_h	t_t	t_b
20 mm	4 mm	25 mm	5 mm	5 mm

Fig. 3a. shows face sheets meshed with 8-noded Hex elements (C3D8) and the core meshed with 4-noded Shell elements (S4). The face-sheets are tied together with the core. A non-linear static procedure is used. Fig. 3b. shows one end of the face-sheets fixed and a uniform pressure loading of 150 KPa on the top face. Simulation is run and two specific responses are monitored- the maximum value of Von Mises Stress $\sigma^{VM}{}_{max}$ and the maximum value of the Displacement magnitude U_{max} . The baseline values of responses are shown in TABLE II.

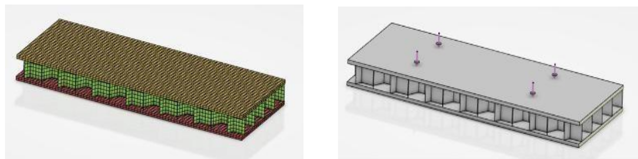


Fig. 3. (a) Mesh and (b) Boundary Conditions

TABLE II. BASELINE VALUES OF RESPONSE VARIABLES

σ_{VMmax}	U_{max}
97.96 MPa	2.97 mm

Design of Experiments (DOE):

A DOE is setup using the Optimal Latin Hypercube technique (OLH). OLH optimizes the combinations of design variables to evenly spread experiment points within n-dimensional space defined by n design variables. Number of levels for each design variable is equal to number of experiment points. This allows for many more points and more combinations to be studied for each design variable. The experiment points are spread evenly, allowing higher order effects to be captured [3]. Fig. 4. shows an example of the sampling using OLH technique in design space.

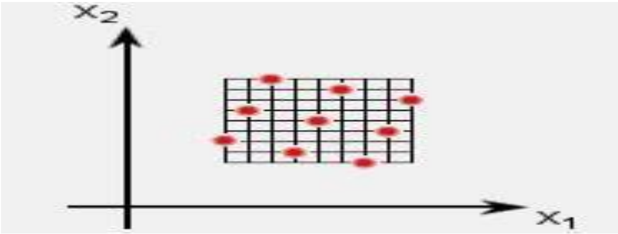


Fig. 4. Design space sampling in OLH technique

This DOE is configured with 5 Design Variables, 2 Response Variables and runs 45 experiment points as shown in TABLE III.

TABLE III. DOE CONFIGURATION

Design Variables (DVs)			
Design Variable	**Lower Bound (mm)**	**Baseline (mm)**	**Upper Bound (mm)**
tb	1	5	5
Cr	5	20	30
tc	1	4	4
Ch	10	25	25
tt	1	5	5
Response Variables (RVs)			
Response Variable	**Objective**		**Weight**
U_{max}	Minimize		1
σ_{VMmax}	Minimize		1

Approximations:

Once the DOE generates the design data set, it is used to test three approximation methods. The methods compared are Response Surface Method, Radial Basis Functions and Universal Kriging method.

Each approximation model is trained and validated to the DOE design data set and the fit measures are noted for both the response variables. Each of the approximation models is validated using the K-fold cross validation method (K=9).

K-Fold Cross Validation:

This validation technique randomly selects data points from the data set and divides the data set into 'K' subsets (folds) of equal size. Each subset of data is removed from the data set, and the model is re-trained using the reduced data set. The cross-validation percent error is then calculated by comparing the actual and predicted values of the response variable at each point that was removed. This procedure is repeated for each of the 'K' subsets [3].

Response Surface Model (RSM):

The Response Surface Model uses a polynomial combination of vectors representing the input parameters. The order of the polynomial regression model depends on the number of experiment points in the data set [3]. This study uses a polynomial with tenth-order uni-variate terms and fifth-order cross terms for the model.

Radial Basis Function (RBF):

The Radial Basis Function model is a type of neural network employing a hidden layer of radial units and an output layer of linear units. The RBF model has a short initialization time and is generally faster than the response surface model for a large number of data points [3].

Universal Kriging (UK):

The Universal Kriging model is an interpolation method that converts partial observations of a spatial field to predictions of that field at unobserved locations. The model is useful in predicting temporally and spatially correlated data and typically creates a good approximation in cases with a small number of data points.

Kriging model is very flexible and provides a choice between a wide range of correlation functions for building the model. Depending on the choice of the correlation function, the model can either honor the data (providing an exact interpolation of the data) or smooth the data (providing an inexact interpolation). [3]

Prediction:

In order to test how the trained and validated models perform in prediction, five random test points are selected. These points are within the bounds of the design space and are points which have not been run previously using FEA. For each test point, a FEA simulation is run and corresponding values of the response variables are noted. These values are then compared with predicted responses from each of the approximation models. TABLE IV shows the five test points.

TABLE IV. TEST POINTS

Test Point	**t_b (mm)**	**C_r (mm)**	**t_c (mm)**	**C_h (mm)**	**t_t (mm)**
1	3.2	5.7	2.4	21	3.2
2	1.5	20	3	20	1.5

3	2.5	15	1.2	14	2.5
4	2.7	10	3.3	11	3.5
5	4.2	12	3.8	18	1.2

TABLE V shows the training measures and the validation measures for approximation using Response Surface Model for both the response variables. Fig. 5. Shows the corresponding fit measures for U_{max} and Fig. 6. shows the corresponding fit measures for σ^{VM}_{max}.

Results and Conclusions

TABLE V. FIT MEASURES FOR RSM

Results for Response Surface Model

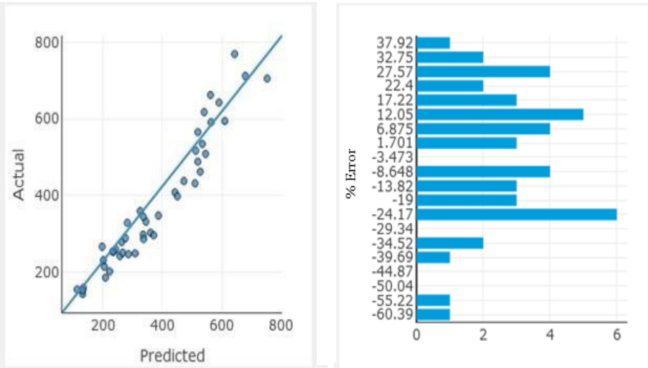
Response Surface Model 1				
Poly Order=10	**Training s measure**		**Validation measures**	
	U_{max} (mm)	σVM_{max} (MPa)	U_{max} (mm)	σVM_{max} (MPa)
R-squared	0.986	0.928	0.933	0.718
R-squared adjusted	0.985	0.919	0.924	0.682
RMS Error	0.027	0.073	0.059	0.144
Residual/Predicted	0.146	0.32	0.391	0.569
% Error Abs Mean	5.44	9.97	11.6	20.3
% Error Abs Max	22.4	33.6	47	60.4
% Error Std Dev	4.63	6.9	10.7	12.7

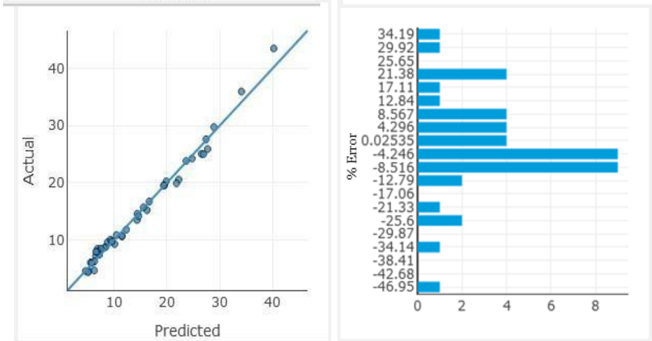
Results for Radial Basis Function

TABLE VI shows the training measures and the validation measures for approximation using Radial Basis Functions for both the response variables. Fig. 7. Shows the corresponding fit measures for U_{max} and Fig. 8. shows the corresponding fit measures for σ^{VM}_{max}.

TABLE VI. FIT MEASURES FOR RBF

Fig. 5. RSM fit measures for U_{max} (a) Actual vs Predicted for

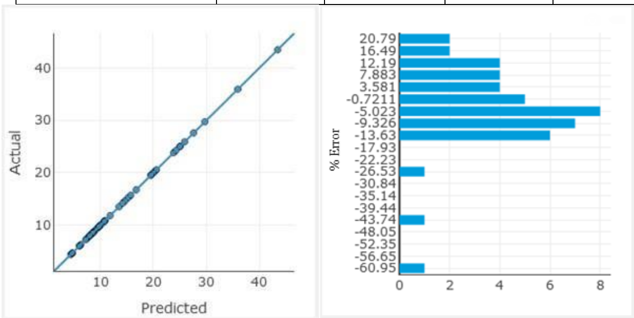




training model (b) Cross Validation % Error

Fig. 6. RSM fit measures for σ^{VM}_{max} (a) Actual vs Predicted for training model (b) Cross Validation % Error

Radial Basis Function 1				
	Training measures		**Validation measures**	
	U_{max} (mm)	σ_{VMmax} (MPa)	U_{max} (mm)	σ_{VMmax} (MPa)
R-squared	1	1	0.938	0.744
R-squared adjusted	1	1	0.93	0.711
RMS Error	1.04E-14	8.65E-15	0.0563	0.137
Residual/Predicted	2.86E-14	2.35E-14	0.392	0.574
% Error Abs Mean	2.15E-12	1.22E-12	10.5	18.5
% Error Abs Max	6.40E-12	3.05E-12	61	61.7
% Error Std Dev	1.78E-12	7.70E-13	10.9	13.4



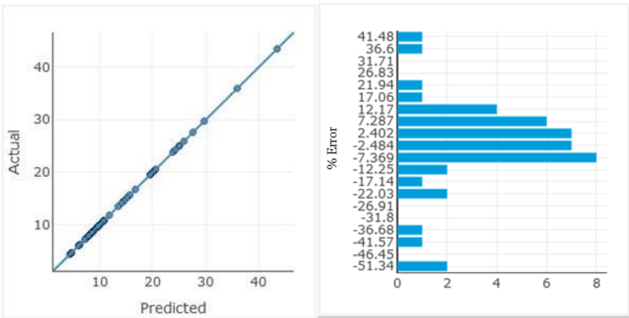


Fig. 7. RBF fit measures for U_{max} (a) Actual vs Predicted for training model (b) Cross Validation % Error

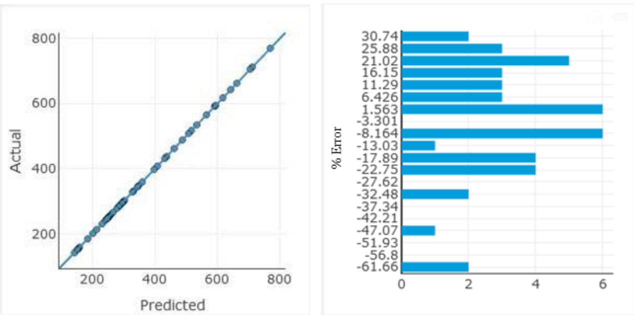


Fig. 9. UK fit measures for U_{max} (a) Actual vs Predicted for training model (b) Cross Validation % Error

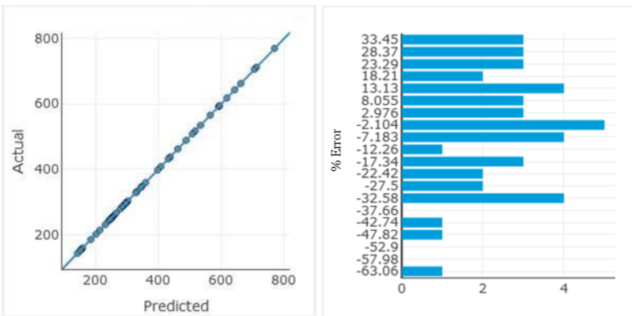


Fig. 8. RBF fit measures for σ^{VM}_{max} (a) Actual vs Predicted for training model (b) Cross Validation % Error

Results for Universal Kriging method:

TABLE VII shows the training measures and the validation measures for approximation using the Universal Kriging method for both the response variables. Fig. 9. Shows the corresponding fit measures for U_{max} and Fig. 10. shows the corresponding fit measures for σ^{VM}_{max}.

Fig. 10. UK fit measures for σ^{VM}_{max} (a) Actual vs Predicted for training model (b) Cross Validation % Error

TABLE VII. FIT MEASURES FOR UK

Universal Kriging 1				
	Training measures		Validation measures	
	U_{max} (mm)	σVM_{max} (MPa)	U_{max} (mm)	σVM_{max} (MPa)
R-squared	1	1	0.907	0.715
R-squared adjusted	1	1	0.895	0.679
RMS Error	6.12E-16	1.31E-15	0.0691	0.144
Residual/Predicted	1.96E-15	1.58E-15	0.364	0.667
% Error Abs Mean	1.30E-13	2.07E-13	12.7	19.6
% Error Abs Max	4.41E-13	3.70E-13	51.3	63.1
% Error Std Dev	1.01E-13	7.06E-14	13.6	14

Comparison of Predicted Response and FEA response

For the 5 chosen test points, TABLE VIII shows the comparison of relative error between predicted U_{max} response and FEA U_{max} response for all approximation models.

TABLE VIII. PREDICTED VS FEA (U_{MAX})

Test Point	U_{max} (mm)				% Error		
	FEA	RSM	RBF	UK	RSM	RBF	UK
1	6.56	4.6	6.58	7.84	30%	0%	20%
2	17.53	19.23	18.81	16.65	10%	7%	5%
3	19.38	21.86	20.98	21.57	13%	8%	11%
4	20.16	20.65	20.19	24.72	2%	0%	23%
5	13.76	13.73	13.54	16.31	0%	2%	19%

For the 5 chosen test points, TABLE IX shows the comparison of relative error between predicted σ^{VM}_{max} response and FEA σ^{VM}_{max} response for all approximation models.

Test Point	σVM_{max} (MPa)				% Error		
	FEA	RSM	RBF	UK	RSM	RBF	UK
1	173.77	143.55	202.68	292.74	17%	17%	68%
2	381.78	551.83	547.5	514.78	45%	43%	35%
3	366.18	434.34	414.41	504.7	19%	13%	38%
4	362.64	333.99	348.76	456.93	8%	4%	26%
5	492.74	515.78	477.91	477.91	5%	3%	3%

TABLE IX. PREDICTED VS FEA (Σ_{MAX}^{VM})

TABLE X shows the comparison of relative error averaged for the 5 test points for both responses and for all the approximation models.

TABLE X. COMPARISON OF AVGERAGE RELATIVE ERROR

Approximation Model	Avg % Error U_{max}	Avg % Error σ^{VM}_{max}
RSM	11	18.8
RBF	3.4	16

UK	15.6	34

For each approximation model, displacement prediction is better than the Von Mises stress predictions for the chosen test points. RBF model seems to have better accuracy in predicting the response variables when compared with the other two models.

Next Steps

The choice of the parameter values for the designs that the model is built with has a considerable impact on the accuracy of the predicted output [4]. The potential next steps could be studying the effect of different sets of design points and the number of experiment points available for training the approximation model on the accuracy of final response predictions. A few other factors that can be considered are the sampling algorithm used in the DOE and the effect of nonlinearities in the underlying baseline FEA

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